

Commentationes

Localized Charge Distributions

VII. Transferable Localized Molecular Orbitals for Acyclic Hydrocarbons

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A localized molecular orbital has been found to extend slightly and regularly into regions away from the chemical bond which contains most of its charge cloud. This was made the basis for a method of transferring localized orbitals among similar molecules. Each localized orbital induces a set of so-called molecule invariant fragments consisting of one bond fragment and collections of geminal fragments, vicinal fragments, and third and fourth neighbor fragments. Localized orbital expansion coefficients in a hybrid basis can be calculated for these molecule invariant fragments without solving any equations or performing any laborious computations.

The present work is an application to acyclic hydrocarbons. The results are based on the analysis of 33 INDO-SCF molecular orbital wavefunctions in the localized representation. Computational methods for obtaining close approximations to localized orbitals are also discussed. The application of a suggested pseudo-eigenvalue localization method and its accompanying self-consistent iteration process are found to not converge.

Key words: Localized charge distributions – Acyclic hydrocarbons

1. Introduction

This work combines two techniques: rigorous molecular quantum mechanics and classical valence theory. The former provides a firm theoretical basis for investigation and counteracts one's own prejudices and inconsistencies, while the latter furnishes the concepts, suggestiveness and subjective intuition so successfully employed even in the years before quantum mechanics [1–4] and certainly before very complete theoretical investigations were available.

The combination is performed with the energy localized representation [5–8] of closed shell INDO [9–13] molecular orbital theory. It – like all others – is a subjective representation, as has been known for many years [14], but is nonetheless rigorous and has proved exceedingly useful for studying phenomena

which do not involve electronic excitations. The present applications will be to the electron distributions in acyclic hydrocarbons, with emphasis on the inherent delocalization and transferability of the localized orbitals which describe the electronic motion. Using INDO overcomes the economic and technical problems (computer storage and time, multicenter integrals, etc.) encountered in *ab initio* methods, which would restrict the study to just a few molecules, at the expense of having a more phenomenological description.

Model geometries are employed [15], in keeping with the model localized orbitals we shall obtain. The orbitals will be described in a basis set of hybrid functions which provide perfectly localized first approximations to them, and hence display their behavior very effectively.

Unlike some studies which concern themselves with the examination of properties [16–19], the present investigation will concentrate on the wave functions themselves. Also unlike some other studies, the present approach does not consider localized orbitals to be perfectly confined to one or two centers [20–24] and examines transferability in much greater detail. The former obtains because of the rigorous approach used and differs from the simple ideas of classical valence theory, while the latter makes transferability as quantitative as is reasonably possible, illustrating its use and theoretical meaningfulness, confirming what has long been appreciated (or suspected) by chemists.

The major effort will be the analysis of the localized orbitals for the thirty-three acyclic hydrocarbons studied, and the extension of the results to cases not treated explicitly. The orbitals will be partitioned into fragments that are molecule-independent, and each group will be classified and discussed separately, revealing their behavior and the ways in which each transfers. Methods for obtaining the unoccupied and canonical symmetry molecular orbitals will also be given.

2. Geometry and Hybridization

2.1. Introduction

Directed valence theory [25, 26] is well-suited for exploiting the intimate connection between molecular geometry and hybridization in organic chemistry. Thus, when carbon atoms are called tetrahedral (alkyl), trigonal (vinyl), or linear (ethynyl), the nomenclature describes the geometric arrangement of the near neighbors and, partially, the hybridization. The latter is only partially specified for three reasons:

1. The relative status of the sigma and pi orbitals is arbitrary.
2. The *s*-character [27] of linear carbon hybrids is arbitrary.
3. The orientation of triple bonds is not defined, i.e. they may be rotated freely about the bond axis.

A dividend from the present analysis is that these hybridization ambiguities can be settled by reference to the localized orbitals (LMO's) that will be presented later. In Case 1, the LMO's mix the sigma and pi symmetries in specific proportions. In Case 2, best fit with LMO's obtains if the *s*-character is assigned the value 0.4779 (it is 0.5000 for pure *sp* hybrids). In Case 3, the orientation can be defined by maximizing overlaps between hybrids and LMO's.

2.2. Bond Skeleton and Numbering of Atoms and Hybrids Basis Functions

Figure 1 depicts the molecules for which we have explicitly calculated LMO's. The atoms are labeled $C_i, C_j, \dots, H_k, H_l, \dots$. The numbers next to each atom label the (hybrid) AO's contributed by that atom. E.g. in the first molecule, propane, the (tetrahedral) hybrids on the carbon atom C_2 carry the identification numbers 5, 6, 7, 8, and the hydrogen $1s$ orbital on H_8 is labeled number 17. Thus each single bond exhibits at its two ends the labels of its two constituent (hybrid) AO's. Double and triple bonds are described by indicating the four or six hybrids that are used to form the two or three banana bonds. Thus the hybrids 3, 4, on C_1 and 7, 8, on C_2 form a double bond in ethylene and the hybrids 2, 3, 4, on C_1 and 6, 7, 8, on C_2 form a triple bond in acetylene.

The cartesian coordinates for these molecules were calculated with the "Model-Builder" program [15]. Bondlengths representing averaged experimental values are assigned automatically by the program, making it especially convenient. These values are basically those adopted in organic chemistry models.

2.3. Pair Equivalent Hybrids in Mutually Perpendicular Planes

These consist of two pairs of equivalent hybrids whose centroids lie in mutually perpendicular planes. They will be designated $g(+), g(-)$ and $h(+), h(-)$, with the former lying in the xz plane and the latter in the yz plane. If A and B are the angles measured from the positive and negative z axes, respectively, toward the

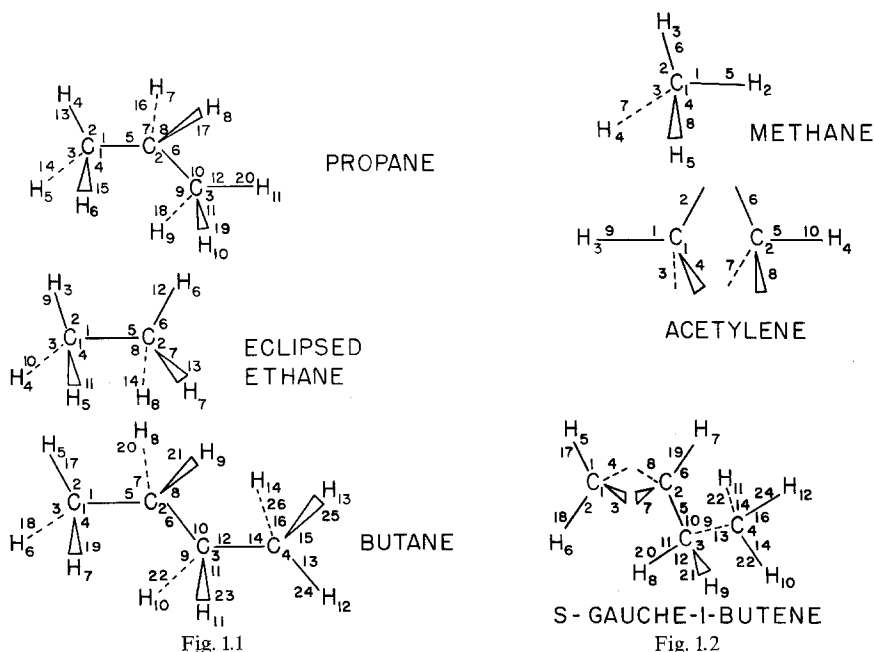


Fig. 1. Molecular bond skeletons and numbering of atoms and basis functions

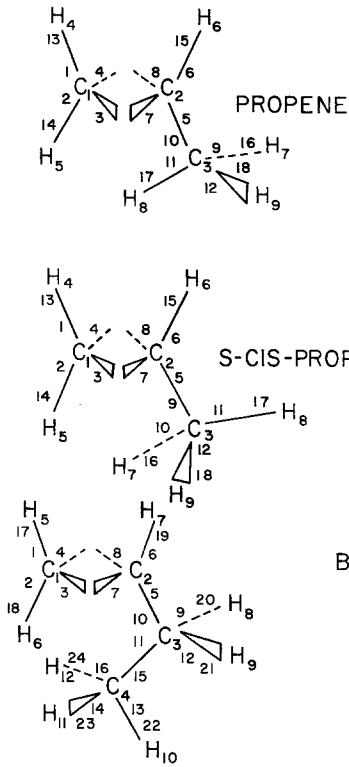


Fig. 1.3

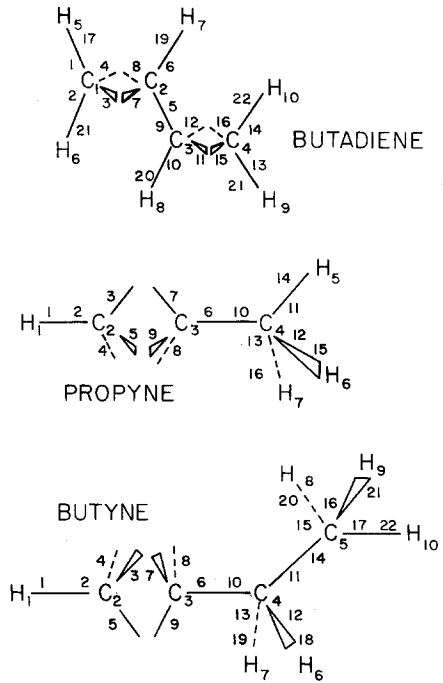


Fig. 1.4

positive x and y axes, respectively, in a right-handed system, then

$$g(\pm) = (s) \cos Q + \sin Q [(z) \cos A \pm (x) \sin A], \quad (1)$$

$$g(\pm) = (s) \cos R + \sin R [(z) \cos B \pm (y) \sin B], \quad (2)$$

where (s) , (x) , etc. refer to the $2s$, $2px$, etc. AO's on carbon. Since there are *four* parameters, but only *three* constraints (normalization is automatic)

$$\langle g(+) | g(-) \rangle = 0, \quad (3)$$

$$\langle h(+) | h(-) \rangle = 0, \quad (4)$$

$$\langle h(\pm) | g(\pm) \rangle = 0 = \cos Q \cdot \cos R + \sin Q \cdot \sin R \cdot \cos A \cdot \cos B, \quad (5)$$

there is one free parameter and hence one additional constraint may be imposed. The explicit relations that result from Eqs. (3)–(5) for the parameters Q , R , A , B are conveniently displayed as

$$(\sin Q \cdot \sin A)^2 = \frac{1}{2}, \quad (6)$$

$$(\sin R \cdot \sin B)^2 = \frac{1}{4}, \quad (7)$$

$$\cos^2 Q + \cos^2 R = \frac{1}{2}, \quad (8)$$

$$\cot^2 A + \cot^2 B = 1. \quad (9)$$

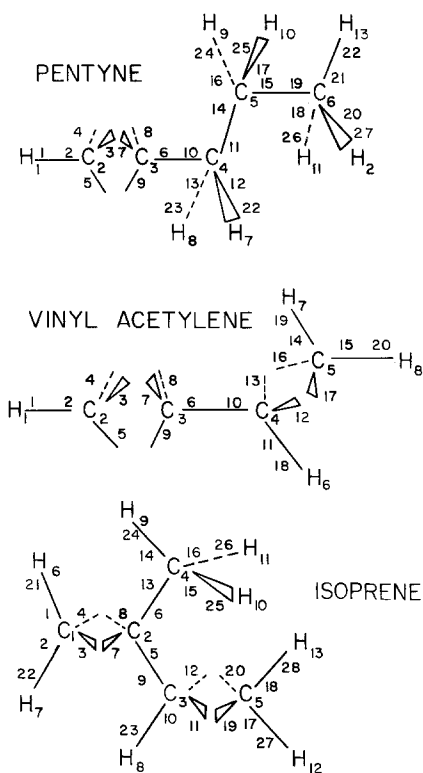


Fig. 1.5

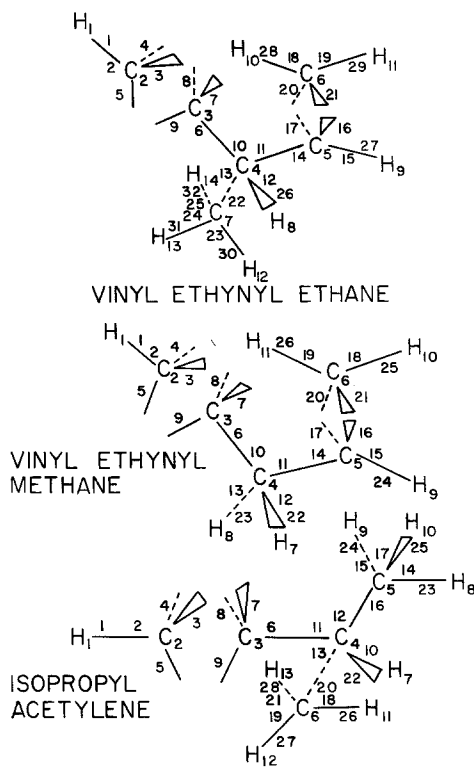


Fig. 1.6

Equations (6) and (7) relate the s character and angular dependence in the *same* hybrid, Eq. (8) connects relative s characters and Eq. (9) connects relative angles. It is straightforward to adjust the two hybrid pairs to specific situations with these equations, in particular to the two cases we are interested in: tetrahedral carbon and trigonal carbon.

2.3.1. Tetrahedral Carbon (Four Single Bonds)

These are obtained by using the additional constraint to fix the angle between all centroids at 109.5° (the tetrahedral value), i.e. by setting

$$A = B \quad (10)$$

so that from Eq. (9)

$$\cos A = \sqrt{3}/3. \quad (11)$$

The remaining parameters follow from Eqs. (6) and (7), so

$$g(\pm) = 0.5(s) + 0.5(z) \pm 0.7071(x), \quad (12)$$

$$h(\pm) = 0.5(s) - 0.5(z) \mp 0.7071(y). \quad (13)$$

These are the well-known sp^3 hybrids.

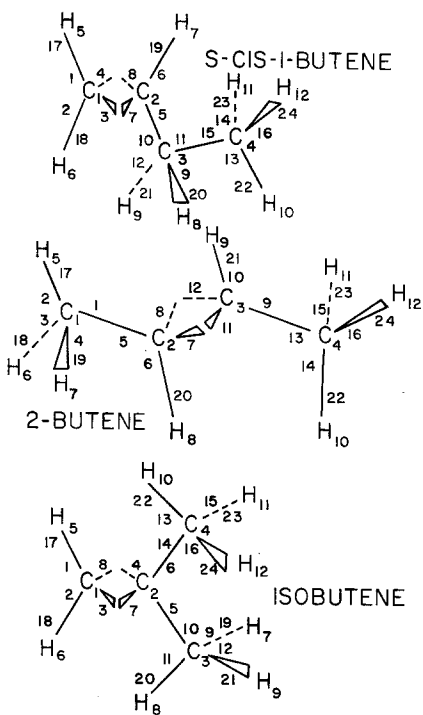


Fig. 1.7

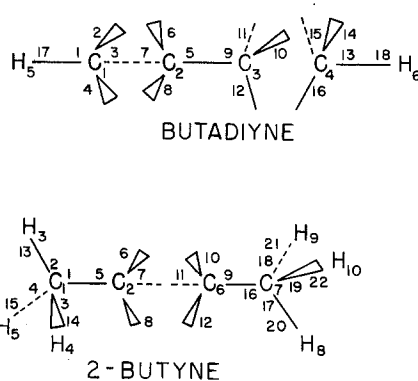


Fig. 1.8

2.3.2. Trigonal Carbon (Two Single Bonds and One Double Bond)

Unlike the tetrahedral, the g 's and h 's are inequivalent here. Suppose the g 's point in the direction of the single bonds and the h 's are the bent banana hybrids pointing into the double bond. Thus

$$A = 60^\circ \quad (14)$$

which implies that

$$B = 51^\circ, \quad (15)$$

giving

$$g(\pm) = 0.5773(s) + 0.4082(z) \pm 0.7071(x) \quad (16)$$

and

$$h(\pm) = 0.4082(s) - 0.5773(z) \mp 0.7071(y). \quad (17)$$

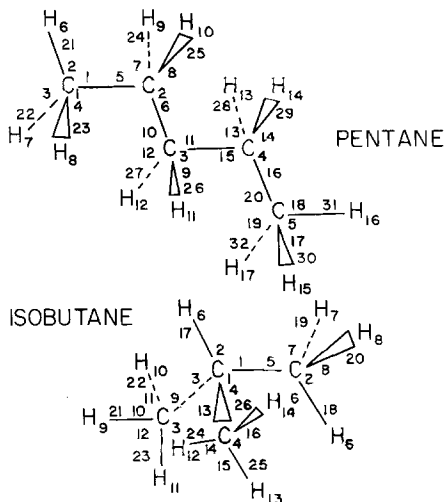
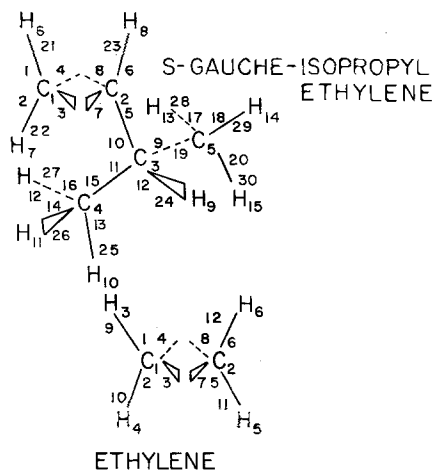
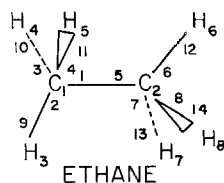
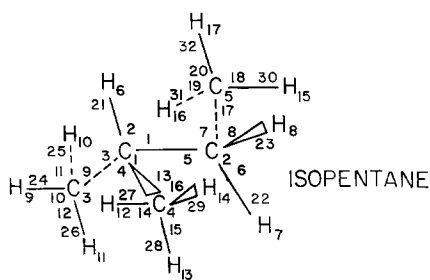
The g 's are the usual sp^2 hybrids, while the h 's are sp^5 , with centroids 50° above and below the bond skeleton. The g 's, being single bond hybrids, are always coplanar with vicinal single bond neighbors. This forces the centroids of the h 's to lie about five degrees away from exact coplanarity with their vicinal bond neighbors.

2.4. Ethynyl Carbon (Single Bond and Triple Bond)

2.4.1. Construction of the Hybrids

Let p , $p(i)$, $i = 1, 2, 3$ be four normalized linear combinations of p -orbitals satisfying

$$\langle p | p(i) \rangle = 0 \quad (18)$$



ETHYLENE

ISOBUTANE

Fig. 1.9

Fig. 1.10

and

$$\langle p(i) | p(j) \rangle = -\frac{1}{2}, \quad i \neq j \quad (19)$$

i.e., the $p(i)$ lie separated by 120° in a plane orthogonal to p . From p and the $2s$ AO can be formed "left" and "right" hybrids which point directly at the neighbor atoms

$$l = (s) \cos Q - (p) \sin Q, \quad (20)$$

$$r = (s) \sin Q + (p) \cos Q. \quad (21)$$

The orthogonal, trigonal banana hybrids are formed from the r and the $p(i)$

$$t(i) = (r + p(i) \sqrt{2}) \sqrt{3}/3. \quad (22)$$

They point into the triple bond and are orthogonal to the hybrid (l) that points into the single bond. Since all of these hybrids are automatically orthogonal, Q is a free parameter. It can therefore be adjusted such that the hybrid l fits as closely as possible the atomic s and p contributions to the actual localized MO's representing the single bonds next to triple bonds. The thirteen cases considered in this study gave

$$l = 0.6736(s) - 0.7391(p) \quad (23)$$

with deviations the same as those for the other single bonds (sp^3 and sp^2). Note that l is not an exact sp hybrid.

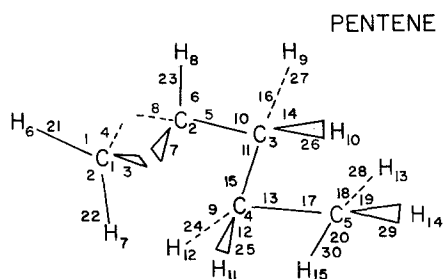
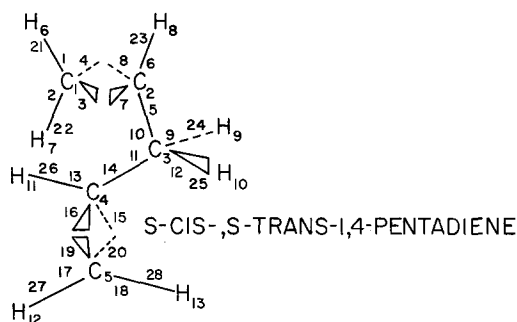


Fig. 1.11

S-TRANS-ISOPROPYL ETHYLENE

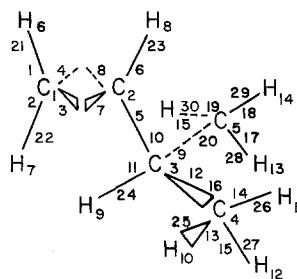


Fig. 1.12

The expression for l implies the expression

$$t(i) = 0.4267(s) + 0.3889(p) + 0.5773 p(i) \quad (24)$$

for the banana hybrids pointing into the triple bond. These are approximately sp^5 , and form an angle of about 63° with the bond axis. It is striking that these hybrids form almost the same angle with each other as those pointing into the double bonds, namely 101° .

2.4.2. Absolute Orientation

Since the ultimate goal is to recognize the similarities among localized orbitals, the free rotation of the $t(i)$ should be removed as consistently as possible. The method chosen was to maximize the overlap between a $t(i)$ and its contribution to one of the banana bond LMO's. Suppose l and r lie along the z -axis, and let the contributions from the perpendicular (x,y) plane to the three LMO's in the triple bond be given by

$$L(i) = a(i) \cdot (x) + b(i) \cdot (y), \quad i = 1, 2, 3. \quad (25)$$

The pair of Eqs. (18) and (19) can be expressed as

$$p(i) = -(x) \sin(W + (i-1)\pi/3) + (y) \cos(W + (i-1)\pi/3) \quad (26)$$

and the variation in W describes the rotation of the hybrid triple around the bond axis. Optimal fit of $p(i)$ to $L(i)$ is therefore obtained from the condition

$$d/dW[\langle L(i)|p(i)\rangle] = 0 \quad (27)$$

which yields an angle $W(i)$ given by

$$\tan(W(i)) = -a(i)/b(i). \quad (28)$$

Three values of $W(i)$ are obtained in this way, from which the weighted mean is formed

$$W = \sum_i W(i) \langle L(i)|p(i)\rangle / \sum_i \langle L(i)|p(i)\rangle. \quad (29)$$

The absolute orientation of the $p(i)$, and hence the $t(i)$, about the bond axis is then obtained by inserting this value of W into Eq. (26).

Even with this procedure, the triple bonds presented situations not encountered with the others. Because of very slow localization and/or slight molecular asymmetry, their centroids were sometimes not coplanar with those of vicinal single bonds. In the discussion of fragments that will be given later, we have therefore extrapolated these exceptions to coplanar orientations.

3. Energy Localized MO's

3.1. Introduction

Energy localized MO's (LMO's) exploit the invariance of the MO equations to unitary transformations, as was pointed out by Fock [14]. The LMO's were themselves proposed later by Lennard-Jones and Pople [28], but remained largely conceptual, even after the advent of computers, until the algorithm of Edmiston and Ruedenberg [5, 6] allowed their practical determination. In two papers including some diatomic and small polyatomic localizations [6, 7], the authors convincingly demonstrated that these were indeed rigorous quantum mechanical analogues to the bonds and lone pairs of chemical intuition, as had been hoped all along.

3.2. Localization Equations

At first, the formal discussion of localized orbitals was confined to equivalent MO's, those which carry the regular representation of the symmetry group, i.e., they are permuted among each other by the group operations. In passing, it was also mentioned that they presumably maximize the self-repulsion sum (localization sum)

$$D = \sum_u [u^2|u^2] \quad (30)$$

over all occupied MO's, where

$$[u|v] = \iint d^3x d^3z u(x)v(z)/|x-z|. \quad (31)$$

Edmiston and Ruedenberg [5] extended these ideas to general systems (i.e., the symmetry requirements necessary for equivalent orbitals were dropped) and adopted the maximization of D and the resulting "localization criteria"

$$[u^2 - v^2|uv] = 0 \quad (32)$$

as the central features for the LMO determinations. This was done by series of successive 2×2 transformations rather like the Jacobi diagonalization method. A remarkable feature of this method is that it has always converged as far as is known, even in those cases where it has led to some ambiguities [6, 7]. These are readily understood [29].

3.3. Semiempirical Energy Localization

In principle, there are as many of these as there are semiempirical theories. Only several have appeared, however: a set of pi orbital localizations [30–32] which used the Mulliken approximation for electron repulsion integrals; and all valence electron localizations using CNDO [33] and INDO [34].

One might wonder what differences obtain between the last two approximations. It has been shown [34] that the retention of the one-center two-electron integrals by INDO gives much better agreement with existing *ab initio* LMO's (INDO LMO's are much closer to the *ab initio* than to the CNDO: in particular, INDO reproduces the *ab initio* banana orbitals while CNDO does not mix the sigma and pi orbitals). Thus, semiempirical LMO's are sensitive to the method of approximating the electron repulsion integrals. This becomes especially important when topics such as hybridization are analyzed, and dictates the use of INDO for studying the paraffins.

3.4. Starting Orbitals

The starting orbitals for the energy localization can be selected in a way that is similar to that used for the Hückel orbitals [30], and is possible for two reasons:

1. The canonical closed shell INDO MO's are readily available.
2. The LMO's in these paraffins are largely two-center "bonds".

Table 1. Localized, least mean square and perturbation MO's in ethane

	LMO 1	SMO 1	PMO 1	LMO 2	SMO 2	PMO 2
1	-108	-58	-83	7069	7070	7069
2	140	140	157	104	54	76
3	6984	6984	6980	104	54	76
4	140	140	157	104	54	76
5	8	57	84	7069	7070	7069
6	172	173	176	104	54	76
7	172	173	176	104	54	76
8	-462	-461	-486	104	54	76
9	-123	-123	-140	-5	-54	-79
10	7115	7115	7114	-5	-54	-79
11	-123	-123	-140	-5	-54	-79
12	-158	-159	-159	-5	-54	-79
13	-158	-159	-159	-5	-54	-79
14	441	441	464	-5	-54	-79

We adopt the following procedure:

1. For each bond, form a perfectly homopolar two-center orbital $(\chi + \chi')/\sqrt{2}$, where χ and χ' are the two hybrids pointing into the bond.
2. Project each of these two-center orbitals into the space of the occupied canonical MO's.
3. Symmetrically orthogonalize these projections.

The detail of this approach have been given elsewhere [30]. The starting localized MO's (SMO's) obtained in this way turn out to be close approximations to the exact LMO's. This is shown in Table 1 for ethane, which lists the coefficients of the two (CH and CC) orbitals in the hybrid basis. The hybrid basis orbitals are the row labels and are taken from the previous Fig. 1, while here and elsewhere the actual coefficients are obtained from those in the tables by multiplying times 10^{-4} . The degree of approximation is typical.

3.5. LMO's as Two-Center Orbitals Perturbed by Bond-Bond Interactions

In the same context, it is of interest to compare a theory of Pople and Santry [36–38] with energy localization. These authors applied the perturbation theory of Coulson and Longuet-Higgins [39–42] to some simple paraffins by using a hybrid basis analogous to the present one and assuming that the unperturbed orbitals were two-center bonds. Only one-electron core energies and bond-bond interactions were included, and although the method strictly provides just the population matrix, coefficients can be extracted from it in several reasonable ways, all of which give about the same results. In order to make the comparison as favorable and simple as possible, the actual LMO's were truncated to two centers and used as the unperturbed functions with the appropriate INDO one-electron matrix elements.

The results (PMO's) for ethane are also shown in Table 1, and again are typical of the other molecules. The agreement with the LMO's is similar to that obtained for the SMO's and thus provides another interesting interpretation: LMO's can be thought of as arising from perturbed, perfectly two-center orbitals. This has two consequences:

1. It gives particular motivation to such concepts as "delocalization" which will be applied later.
2. The PMO's anticipate some of the dependences on relative bond orientations, types and distances [36–38] that play a great role in the analysis.

3.6. Energy Localized Orbitals from Pseudo-Eigenvalue Equations

Ruedenberg [8] has shown that LMO's satisfy the pseudo-eigenvalue equations (see also the work of Gilbert [43], which is closely related)

$$(F + L) |n\rangle = \gamma_m |n\rangle, \quad n = 1, \dots, N \quad (33)$$

where F is the Fock operator,

$$L = \sum_{i \neq k}^N \sum_k^N |i\rangle (C_{ik} - \gamma_{ik}) \langle k|, \quad (34)$$

with

$$\gamma_{ik} = \langle i|F|k\rangle \quad (35)$$

and

$$C_{ik} = |\langle ii - kk|ik\rangle|. \quad (36)$$

It was hoped that LMO's could be obtained with an SCF iteration process.

The SMO's provided an interesting chance to test this, being very close to the LMO's and thus presumably requiring few iterations for convergence. This was not the case however, for except for ethane, all attempts at iterative solution of Eq. (34) failed: the C_{ik} , though small initially (for propane the largest were around 0.0025 a.u.), did not become successively smaller.

3.7. The LCAO Expansions

The complete LCAO LMO's for each molecule are presented in Table 2. The expansion coefficients are given in the previously described hybrid basis with units of $(\text{a.u.})^{-3/2} \cdot 10^4$. Expansions in a Slater orbital basis will be provided upon request. The rows are labeled by the numbers assigned the hybrids in Fig. 1. The LMO's are also labeled by numbers and these form the column headings. The detailed discussion of these LMO's will be given later in terms of molecule-independent (MI) coefficients.

Localization was terminated when an iteration [each set of $N(N-1)/2$ transformations] left D stable to ten decimals. It was found that even in the worst cases this gave coefficients stable to several parts in ten in the fourth decimal place. Due to the development of a first order method of energy localization [35] toward the end of the work, the usual Edmiston-Ruedenberg scheme was used. This normally took fifteen minutes or less on an IBM 360/50. but needed more than thirty minutes for some of the larger or triply bonded molecules.

4. Molecule Invariant LMO Fragments

4.1. Local Delocalization and LMO Fragments

It is clear from the LCAO expansions in the hybrid basis (Table 2) that the LMO's are *not* confined to two centers. This fact is related to the occurrence of long range bond orders, which was pointed out by Pople and Santry [36–38], but seems not to have been widely appreciated, perhaps because such delocalization appears intuitively unappealing. However, recent studies from this laboratory have revealed the importance of such "local delocalization" in many contexts: the origin of aromatic stability is due to geminal delocalization [30–32], and the origins of rotation barriers [44–47] as well as the systematic interpretation of bond dipole moments and energies [18] is intimately connected with vicinal delocalization. There are therefore good reasons to devote an essential part of the present study to the delocalization of the LMO's.

Local delocalization means that each LMO contains contributions from various bond regions. In the present discussion, the term "bond region" will be

used to denote the space lying between two chemically bonded atoms. Each LMO is therefore a sum of several LMO *fragments*, each of which is associated with a specific bond region. The character of these LMO fragments will be the principal objective of the subsequent discussion because, in acyclic paraffins, transferability and regular behavior are most clearly revealed by them. Standard forms for the (LMO) fragments will be catalogued for various conditions in series of tables from which they can be retrieved to construct practically exact LMO's for almost any given acyclic hydrocarbon *without solving any equations*. For those few exceptions that were not explicitly covered by this work, close estimates (to within a few percent) were obtained by sensible perusal of the discussion of related cases.

An LMO fragment is characterized (1) by the LMO to which it belongs and (2) by the bond region in which it is located. We shall discuss both in turn.

4.2. LMO Characteristics of Fragments

The LMO characteristics of a fragment are based on the fact that by far the largest part of each LMO lies in one particular bond region (the coefficients of the hybrids pointing into this region are always about 0.7), so that it is justifiable to say: a particular LMO "essentially describes" the bond in which it is predominantly concentrated. Thus, LMO number three in pentane (Table 2) essentially describes the single bond between the atoms C_1 and H_6 . This principal fragment will be called the *bond fragment* of that LMO, and it will be used to characterize the LMO itself. Thus the LMO number three will be called a "CH bond LMO".

All other fragments of an LMO will be characterized by their location relative to the bond fragment, denoted as *geminal fragments*, *vicinal fragments* and *third* and *fourth neighbor fragments*. Thus in LMO number three of pentane (Table 2), the fragment between atoms C_1 and C_2 is a geminal fragment, the one between C_2 and C_3 is a vicinal fragment, and those between C_3 and C_4 and C_4 and C_5 are third and fourth neighbor fragments, respectively. Higher fragments will not be considered because of their small coefficients.

We adopt the convention that the bond fragment is positive, which implies that, in each LMO, the coefficients of the hybrids in the bond fragment are positive.

4.3. Characterization of LMO Fragments by Bond Regions

The characterization of an LMO fragment with respect to the nature of the bond region in which it lies has to do with the location of that bond region within the total bond skeleton. The character of a bond region, in turn, is determined by the hybrid AO's pointing into it from the two constituent atoms. We start therefore by classifying the character of these hybrids. In the following the term "hybrid" is always meant to include the hydrogen AO, unless stated otherwise.

Hybrid AO's can be located on four types of atoms

- (1) on hydrogen atoms denoted by H;
- (2) on alkyl type carbon atoms ($>C<$) denoted by CA;
- (3) on vinyl type carbon atoms ($=C<$) denoted by CV;
- (4) on ethynyl type carbon atoms ($\equiv C-$) denoted by CE.

The carbon atoms can be further classified according to their "geminal character"

- (1) methanic (bonded to hydrogens only);
- (2) primary (bonded to one other carbon), denoted by CP;
- (3) secondary (bonded to two other carbons), denoted by CS;
- (4) tertiary (bonded to three other carbons), denoted by CT;
- (5) quaternary (bonded to four other carbons), denoted by CQ.

Finally, the carbon hybrids will be characterized by what kind of bond lies in the coplanar position(s) vicinal to them, *cis* or *trans*. These are denoted by

- (1) H, when a CH bond is vicinal;
- (2) C, when a CC bond is vicinal;
- (3) 2, when a double bond is vicinal;
- (4) 3, when a triple bond is vicinal.

The *cis* are distinguished from the *trans* by an asterisk, e.g. H* means the CH bond in the vicinal position lies *cis*.

Thus there exist a considerable number of different hybrid AO's and every bond region can be characterized by the hybrids pointing into it. Since these LMO fragment characteristics will occur again and again in the subsequent discussions, we shall give here a complete description for those that occur in the study, together with the abbreviations to be used in the sequel. Following this, we shall show how the *cis* and *trans* fragment abbreviations are used.

1. CMH = single bond between methanic carbon and hydrogen.
2. CPH = single bond between primary alkyl carbon and hydrogen.
3. CSH = single bond between secondary alkyl carbon and hydrogen.
4. CTH = single bond between tertiary alkyl carbon and hydrogen.
5. CPVH = single bond between primary vinyl carbon and hydrogen.
6. CSVH = single bond between secondary vinyl carbon and hydrogen.
7. CEH = single bond between primary ethynyl carbon and hydrogen.
8. CPCP = single bond between two primary alkyl carbons.
9. CPCS = single bond between primary and secondary alkyl carbons.
10. CPCT = single bond between primary and tertiary alkyl carbons.
11. CPCQ = single bond between primary and quaternary alkyl carbons.
12. CPCSV = single bond between primary alkyl and secondary vinyl carbons.
13. CPCTV = single bond between primary alkyl and tertiary vinyl carbons.
14. CPCE = single bond between primary alkyl and secondary ethynyl carbons.
15. CSCS = single bond between two secondary alkyl carbons.
16. CSCT = single bond between secondary and tertiary alkyl carbons.
17. CSCQ = single bond between secondary and quaternary alkyl carbons.
18. CSCSV = single bond between secondary alkyl and secondary vinyl carbons.
19. CSCTV = single bond between secondary alkyl and tertiary vinyl carbons.
20. CSCE = single bond between secondary alkyl and secondary ethynyl carbons.
21. CTCT = single bond between two tertiary alkyl carbons.
22. CTCQ = single bond between tertiary and quaternary alkyl carbons.
23. CTCSV = single bond between tertiary alkyl and secondary vinyl carbons.
24. CTCTV = single bond between tertiary alkyl and tertiary vinyl carbons.
25. CTCE = single bond between tertiary alkyl and secondary ethynyl carbons.
26. CQCQ = single bond between two quaternary alkyl carbons.
27. CQCSV = single bond between quaternary alkyl and secondary vinyl carbons.

28. CQCTV = single bond between quaternary alkyl and tertiary vinyl carbons.
29. CQCE = single bond between quaternary alkyl and secondary ethynyl carbons.
30. CSVCSV = single bond between two secondary vinyl carbons.
31. CSVCTV = single bond between secondary and tertiary vinyl carbons.
32. CSVCE = single bond between secondary vinyl and secondary ethynyl carbons.
33. CTVCTV = single bond between two tertiary vinyl carbons.
34. CTVCE = single bond between tertiary vinyl and secondary ethynyl carbons.
35. CECE = single bond between two secondary ethynyl carbons.
36. BCPVCPV = one of the double banana bonds between two primary vinyl carbons.
37. BCPVCSV = one of the double banana bonds between primary and secondary vinyl carbons.
38. BCPVCTV = one of the double banana bonds between primary and tertiary vinyl carbons.
39. BCSVCSV = one of the double banana bonds between two secondary vinyl carbons.
40. BCSVCTV = one of the double banana bonds between secondary and tertiary vinyl carbons.
41. BCTVCTV = one of the double banana bonds between two tertiary vinyl carbons.
42. BCPECPE = one of the triple banana bonds between two primary ethynyl carbons.
43. BCPECSE = one of the triple banana bonds between primary and secondary ethynyl carbons.
44. BCSECSE = one of the triple banana bonds between two secondary ethynyl carbons.

We point out the symbols are "symmetric", i.e., CSCP means the same thing as CPCP. The *trans* fragments are described by adding symbols to these, usually preceded by a dash. For example, CPCT-H23 means that a CH bond, a double bond and a triple bond all lie *trans* to the CPCT bond. If the double bond lay *cis*, this would be described as CPCT-H2*3. Many times, the CH will be understood, e.g., the preceding will be written CPCT-2*3.

Finally, we distinguish in each LMO fragment the *near* hybrid and the *far* hybrid, according to their position relative to the bond fragment of the LMO.

5. MI Bond Fragment Coefficients

5.1. General Characteristics

All bond fragment hybrid coefficients have values near those of the ethane CPCP bond fragment, 0.7070, and are an order of magnitude greater than those describing any other fragment types. The bond fragment coefficients are also the only ones that always have the same sign. Coefficient deviations from the ethane CPCP value depend mostly on (1) whether the atoms involved are carbon or hydrogen, and (2) the geminal character of the carbon atoms involved. Generally smaller coefficient variations are transmitted in *planes* including the bond frag-

ment and those fragments lying *cis* or *trans* to it. We term the latter fragments *conjugate fragments*. The variations induced in these planes are determined by (1) the conjugate fragment nature and orientation (relative to the bond fragment), and (2) a partially steric effect involving *certain* bond region chains which lie beyond the conjugate fragment and in the aforementioned plane. This last effect is only partially steric because it transmits only through the plane.

5.2. Tabulation

The bond fragment coefficients for any acyclic hydrocarbon can be calculated from Table 3. We shall illustrate this explicitly for a CTCE bond fragment. Coefficients for other bond fragments are calculated in the same way.

The numbers labeled "Single Bond Fragments" in the table characterize all *single* bond fragments having (1) no conjugate fragments, or (2) CH conjugate fragments in *trans* positions. The tabulated numbers describe the fragment relative to the homopolar CPCP bond fragment in ethane. To pursue our example of CTCE, consider the numbers that appear in the CTX rows and X = CE column. These are -145 and 10. Thus, the CT and CE bond fragment coefficients for any CTCE bond region with a *trans* CH conjugate fragment are given by,

$$CT = CP(\text{ethane}) - 145 = 6925$$

and

$$CE = CP(\text{ethane}) + 10 = 7080.$$

Thus, the sp^3 hybrid coefficient is 0.6925 and sp coefficient is 0.7080. In just the same way, the numbers labeled "Banana Bond Fragments" characterize all double and triple bond regions relative to the CPCP bond fragment in ethane.

Conjugate fragment influences on single and double bond fragments are shown in the two blocks appropriately labeled in Table 3. The row label - in the block for single bonds refers to all single bond fragments, i.e. the influence of a given conjugate fragment is independent of the bond fragment. The row label = in the block for double bonds has a similar meaning, but now refers to double bond fragments. The Ω denotes the conjugate fragments. For example, if we wanted the bond fragment coefficients for a CTCE bond with a CC bond in a *trans* position, we would add the entries in the - rows, $\Omega = C$ column to our last set of numbers, i.e.

$$CT = 6925 - 10 = 6915$$

and

$$CE = 7080 + 20 = 7100.$$

Thus, *conjugate effects are simply additive*. If there were simultaneously a double banana hybrid in a *cis* position, we would have,

$$CT = 6915 + 0 = 6915$$

and

$$CE = 7100 - 15 = 7085$$

for our CTCE bond fragment coefficients.

Conjugate fragment influences on triple bond fragments are slightly different, but are calculated in the same way. As can be seen from the table, there is a greater sensitivity to the type of the conjugate fragment, but *cis* and *trans* conjugate fragments induce the same variations.

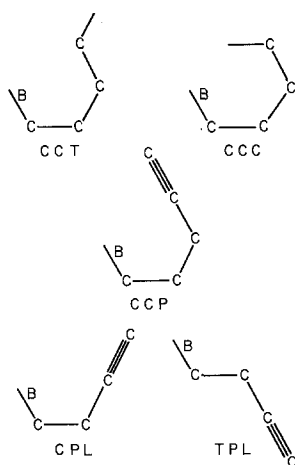


Fig. 2. Conformations transmitting partially steric influences

Conformations for the partially steric influences shown in the table are defined on Fig. 2. The *B* labels the bond fragment. The effects are simply additive and are the same for all bond fragments. For example, suppose the CC conjugate fragment trans to our CTCE bond fragment gives rise to the conformation TPL. We, therefore, take the numbers from column TPL and add them to our last set of coefficients, viz.

$$CT = 6915 + 15 = 6930$$

and

$$CE = 7085 - 10 = 7075 .$$

6. MI Geminal Fragment Coefficients

6.1. General Characteristics

Geminal fragment hybrid coefficient magnitudes are only about 2% as large as those of the bond fragments, and show diverse relative magnitudes and signs. From our experience with bond fragments, we might expect to find classes of CH and CC geminal fragment coefficients that are influenced by the geminal character of their carbon atoms and by the vicinal fragments lying coplanar with the geminal fragment. This is indeed the case. However, a geminal fragment cannot be totally separated from its adjacent bond fragment in the sense that most of the factors which were needed to characterize the bond fragment coefficients have to be repeated to characterize the geminal fragment coefficients. Thus, many combinations of labels are possible, and all are needed to catalog these coefficients.

The greatest geminal fragment coefficient dependences are upon (1) the geminal character of the so-called *pivot* carbon atom (the carbon atom shared by the bond and geminal fragments), and (2) the type and geminal character of the *other* bond fragment and geminal fragment atoms. Superimposed are usually smaller coef-

ficient variations induced by (1) the conjugate fragments, and (2) the fragments lying vicinal and coplanar with the geminal fragment itself. There are also partially steric effects from certain bond region chains which are coplanar with the geminal fragment and lie beyond the *conjugate* fragment. However, we find these to be important only when the geminal fragment is primary vinyl CH.

6.2. Tabulation

As we have pointed out, there are many factors to influence the geminal fragment coefficients, and these can lead to a variety of diverse situations. Thus, to make our tabulation compact, we have used slightly different methods for the cases when the pivot carbon is (1) alkyl, (2) vinyl, (3) ethynyl.

6.2.1. Alkyl Pivot Carbon

All alkyl carbon geminal fragment coefficients can be calculated from Table 4. Reference coefficients for secondary pivot carbons are shown in the first four columns and first eight rows of the block labeled "Alkyl Carbon Reference Coefficients". For this case, the row labels describe secondary geminal fragments having either (1) CH fragments in *trans* vicinal positions, or (2) no coplanar vicinal fragments. The column labels describe the adjacent bond fragments, which have either (1) CH conjugate fragments in *trans* positions, or (2) no conjugate fragments. For example, a CSCP geminal fragment (with a CH fragment lying *trans*) adjacent to a CSCSV bond fragment (with *trans* CH conjugate fragments) has a coefficient of -115 for the sp^3 pivot carbon hybrid and 60 for the primary carbon sp^3 hybrid.

The remaining column labels describe what happens to the secondary geminal fragment coefficients when the *pivot* carbon is changed from secondary to either methyl (M), primary (P), tertiary (T), or quaternary (Q). For example, to obtain coefficients for a CTCP geminal fragment (with CH fragments *trans*) adjacent to a CTCSV bond fragment (with *trans* CH conjugate fragments), we must add the third and fourth rows of column T to the third and fourth rows of column CSCSV, i.e.,

$$CT = -115 + 20 = -95$$

$$CP = 60 - 5 = 55.$$

The block labeled "Far Carbon Atom Influences" describes the coefficient variations which occur when the pivot carbon in the previous block is fixed and the other carbon atoms in the bond or geminal fragment are changed. The first two columns refer to the bond fragment far carbon, and the last two columns refer to the geminal fragment far carbon. In both cases the CA and CV stand for alkyl and vinyl carbons, respectively. The row labels complete the specification of the far carbon atom. For example, the CT rows of the bond fragment CV column label the far atom of the bond fragment as a tertiary vinyl carbon atom (CTV). To illustrate this, suppose we ask what happens to our geminal CTCP coefficients when the geminal far carbon becomes a secondary alkyl and the bond far carbon

Table 4. Alkyl carbon LMO geminal fragment coefficients

PIVOT ALKYL CARBON REFERENCE COEFFICIENTS																		
	GSH	CSGP	CSCSV	CSCE	H	P	T	Q	PIVOT BOND GEMINAL			- n -						
									CP	H	CA	C	2	3	H*	C*	2*	3*
CS	170	140	75	160	-75	-35	40		CP	H	CA	C	2	3	H*	C*	2*	3*
H	-135	-25	-25	0	35	20	-5		CA ^a	≠CV	≠CV	0	-100	-40	-135	-70	-35	-45
CS	-85	-60	-115	-50	-25	20	40		CA	CV	≠CV	0	-25	-10	-40	-30	-5	5
CP	0	65	60	95	10	-5	-10		CA	CV	≠CV	-10	-10	10	0	-35	0	0
CS	-25	-15	-200	-25	20	40			CA	CV	≠CV	0	0	0	-90	0	0	0
CSV	25	90	80	110	10	-5	-10		CA	≠CV	CV	0	0	0	0	0	0	0
CS	-150	-115	-160	-125	-25	20	40		CA	≠CV	CV	-40	-100	-40	-135	-70	-35	-45
CE	0	115	105	140	10	-5	-10		CA ^b	CV	CV	-10	-25	-10	-40	-30	-5	5
												0	0	0	90	20	20	0
												0	0	0	0	-10	0	0

FAR CARBON ATOM INFLUENCES												
	BOND			GEMINAL			PIVOT			BOND GEMINAL		
	CA	CV	CV	CA	CV	CV	CA	≠CV	ALL	CA	CV	CV
CS	-15	0	0	-20	-5							
CT	-30	-10	0	-40	-30							
CQ	0	0	0	-10	0							
	-45	0	0	-60	-15							

^a Except case in first row
^b Cannot be CP

becomes a tertiary vinyl. The changes are described by adding the first two rows of column three (the variations induced by the geminal far atom change) and the second two rows of column two (the variations induced by the bond far atom change) to our previous coefficients, i.e.,

$$CT = -95 - 20 - 10 = -125$$

$$CS = 55 - 5 + 0 = 50.$$

The block labeled “ Ω ” describes what happens when (1) conjugate fragments are introduced or (2) the trans CH conjugate fragments are changed. The first column label characterizes the pivot atom. The next two column labels characterize the far atoms of the bond and geminal fragments, respectively. Remaining column labels classify the conjugate fragment Ω . The row labels CP, CA, CV and H refer to primary alkyl carbons, all alkyl carbons, all vinyl carbons, and hydrogen, respectively. The notation $\neq CV$ means that the atom in question may not be a vinyl carbon. As an example, suppose our CTCS geminal fragment adjacent to a CTCTV bond fragment now has a CH bond lying *cis*. The row groups 5 and 6 of the block characterize this combination of bond and geminal fragment far carbons. Thus we proceed to column H* to obtain our coefficients

$$CT = -125 - 90 = -215$$

$$CS = 50 + 0 = 50.$$

The only remaining influences on alkyl carbon geminal fragment coefficients arise from vicinal fragments lying coplanar with the geminal fragment itself. These are labeled by ω and appear as such in the table. The method of classification is the same as for the Ω (conjugate fragment) influences, but here the only far atom dependence is a CV or $\neq CV$ in the bond fragment. For example, if a CC single bond lay *cis* to our CTCS geminal fragment, the coefficients would be calculated from the C* column of rows 3 and 4, i.e.

$$CT = -215 + 40 = -175$$

$$CS = 50 - 10 = 40.$$

6.2.2. Vinyl Pivot Carbon

These coefficients are given in Table 5. Since the tabulation is very similar to the one just discussed, we shall concentrate on the differences between the two schemes. First of all, notice that the character of the pivot carbon is incompletely specified in the first 10 rows and the first 5 columns of the block labeled “Vinyl Carbon Reference Coefficients”. This is done because, taken together, the row and column labels always completely specify the pivot carbon. For example, taken together the labels for the first 2 rows (geminal fragment) and the first column (bond fragment) can only describe a primary vinyl CH (CPVH) geminal fragment adjacent to a primary vinyl CH (CPVH) bond fragment. That is, the

Table 5. Vinyl carbon LMO geminal fragment coefficients

PIVOT VINYL CARBON REFERENCE COEFFICIENTS																			
	- n -																		
	CVII	CVI	CVCV	CUCV	CUCV	CVCE	BCVWCPV	BCSVCPV	BCVCPV	PIVOT	BOND	GENERAL	C	2	3	H*	C*	2*	3*
CV	290	275	260	275	290	340	375	375	375	CFV	H	H	-10				30		
H	-255	-175	-155	-165	-150	15	-10	-10	-10				-15				-30		
CV	80	105	75	65	115	-40	200	220	220	CSV	H	C	-30	-30	0	0	0	0	0
CP	-150	-65	-50	-50	-40		90	75	75				-15	-15	0	0	0	0	0
CV	75	95	-100	-110	-65	-20	175	170	170	CSV	C	H	15	15	-10	15	15	15	10
CSV	-60	-15	-30	-30	-20		125	120	120				0	0	0	0	0	0	0
CV	85	105	-90	-100	-55	-20	185	180	180	CTV	C	C	-30	-30	0	0	-15	-30	0
CTV	-60	-15	-30	-30	-20		135	130	130				-15	-15	0	0	-15	-15	0
CV	25	35	-10	-20	25		145	85	85	CFV	H	BCV	-20			25			
CE	-110	-65	-75	-75	-65		90	40	40				-20			0			
BCPV	-240					-15				CV	H,C	BCV	0	0	0	0	0	0	0
CFV	-110					-15				CV	BCV	BCV	0	0	0	0	0	0	0
BCSV	-235	-220	-200	-210	-230		50		50	CV	BCV	H	0	0	0	0	0	0	0
CFV	-95	-55	-50	-50	-30		-60		-60	CV	BCV	C	-60	-60	0	-60	-60	-60	0
BCV	-200	-180	-190	-210			115		115	CV	BCV	C	-30	-30	0	-10	-30	-30	0
CFV	-45	-40	-40	-20			-105		-105	CV	BCV	BCV	0	0	0	0	0	0	0

FAR CARBON ATOM INFLUENCES																			
	- n -																		
	CH	CC				BCV				PIVOT	BOND	GENERAL	C	2	3	H*	C*	2*	3*
BOND	GENERAL	BOND	GENERAL	BOND	GENERAL	BOND	GENERAL	CSV ^a	H										
CS	-10	20	-15	-20	-15	-20	20	-15	20	CSV ^a	H	C	-10	-10	20	0	-10	-10	20
	-10	0	0	0	0	0	5	-10	5				0	0	0	0	0	0	0
CT	-20	40	-30	-40	-30	-40	40	-30	40	CSV	C	H	40	40	0	0	0	0	0
	-20	0	0	0	0	0	10	-20	10				-10	-10	30	0	0	0	30
CQ	-30	60	-45	-60	-45	-60	60	-45	60	CTV	C	C	-10	-10	20	0	-10	-10	20
	-30	0	0	0	0	0	15	-30	15				0	0	0	0	0	0	0
CSV	-25	35	30	-20	-20	-20	-20	-5	-5	CFV	H	BCV	0	0	0	0	0	0	0
	-10	0	-15	-20	-20	-20	-5	-5	-5				0	0	0	0	0	0	0
CTV	-50	70	60	-40	-40	-40	-40	-10	-10	CSV	H	BCV	10	10	10	10	10	10	10
	-20	0	-30	-40	-40	-40	-10	-10	-10				-10	-10	20	-10	-10	20	20
										CV	C	BCV	40	40	0	0	40	40	0
										CFV	BCV	H	-10	-10	30	0	-10	-10	30
										CFV	BCV	H	20			-35			0
										CV ^b	BCV	ALL	0	0	0	0	0	0	0

^a The CFV (pivot), H(bond), H(geminal) is covered by the first row of block n
^b Except for the case in the preceding row.

doubly bonded vinyl pivot carbon is singly bonded to 2 hydrogens and thus must be a primary vinyl carbon. Similarly, the first and second pairs of row labels combine with the second column label to describe secondary (CSV) and tertiary (CTV) vinyl carbons, respectively. On the other hand, the pivot carbon is completely characterized in the last three pairs of rows and last three columns in the block: This also serves to characterize it when these labels are combined with those of the first five columns and last three pairs of rows, respectively. For example, a vinyl CH geminal fragment (the CVH rows in the block) adjacent to a banana bond fragment between two primary vinyl carbon atoms (the BCPVCPV column in the block) can only be a primary vinyl CH fragment (CPVH).

The coefficient variations induced by the far carbon are shown in the block labeled "Far Carbon Atom Influences". There are two far carbons to consider, the far carbon of the bond fragment and the far carbon of the geminal fragment. The two cases are distinguished by the "Bond" and "Geminal" in the second row of column labels. Furthermore, the influence of the far carbon is different depending on whether the *adjacent* fragment (bond or geminal) containing the pivot atom is a CH, single CC, or banana CC. The first row of column labels classifies this. Finally, the influence of far alkyl carbons and far vinyl carbons differs. This is described by the row labels.

We shall now consider several examples. First, suppose we have a CSVCP geminal fragment adjacent to a CSVH bond fragment. The coefficients describing these are,

$$\text{CSV} = 80$$

$$\text{CP} = -150$$

as taken from the reference coefficient block. If the far carbon of the geminal fragment is changed to CS, the new coefficients are calculated by adding the numbers in the first two rows (since the new far carbon is CS) of Column 4 (since the far carbon of a geminal CC fragment has been changed) in the far carbon influences block, i.e.,

$$\text{CSV} = 80 - 20 = 60$$

$$\text{CS} = -150 + 0 = -150.$$

Now suppose we have a CTVCP geminal fragment adjacent to a CTVCSV bond fragment so that

$$\text{CTV} = 75$$

$$\text{CP} = -50.$$

If the CSV of the bond fragment becomes CTV, we find the coefficients to be

$$\text{CTV} = 75 + 60 = 135$$

$$\text{CP} = -50 - 30 = -80$$

from rows 9 and 10, Column 3 of the far atom influences. Next, suppose that a CTVCS geminal fragment is adjacent to a BCTVCSV bond fragment. From the set of reference coefficients we find for a CTVCP geminal fragment adjacent to

a BCTVCPV bond fragment

$$\text{CTV} = 220$$

$$\text{CP} = 75.$$

By using rows 7 and 8 of Column 5 in the far atom influences, we find the effect of changing CPV to CSV in the bond fragment to be

$$\text{CTV} = 200$$

$$\text{CP} = 70.$$

Finally, we find the desired coefficients from rows 1 and 2, Column 4, in the far atom block

$$\text{CTV} = 180$$

$$\text{CS} = 70.$$

The primary vinyl CH geminal fragments are the only class for which significant partially steric influences are observed. These are listed in the block of the same name. The column labels of this block locate the coplanar bond chain relative to the *bond fragment*. The bond chain orientations are shown on the previous Fig. 2. For example, if a CPVH geminal fragment is adjacent to a CPVH bond fragment having CCT coplanar chains, its coefficients are

$$\text{CPV} = 290 + 45 = 335$$

$$\text{H} = -255 - 30 = -285.$$

The remaining 2 blocks of the table describe the influence of conjugate fragments, Ω , and the fragments lying vicinal and coplanar with the geminal fragment, ω . Apart from the different sensitivities as manifest by the greater number of rows in each block (relative to the alkyl pivot carbon case), the tabulation is the same as given for alkyl pivot carbons.

6.2.3. Ethynyl Pivot Carbon

The ethynyl pivot carbon reference coefficients are divided into 2 classes, (1) banana geminal fragments adjacent to single-bond and banana bond fragments, and (2) single-bond geminal fragments adjacent to banana bond fragments. These appear in the first and second blocks, respectively of Table 6. The row labels always characterize the geminal fragments and the column labels always characterize the bond fragments. Note that the character of the pivot carbon is usually incompletely specified. The reasons for this are as discussed in the vinyl pivot carbon section.

The remaining blocks are directly comparable to those given for alkyl pivot carbons. This being so, reference should be made to the alkyl pivot carbon section for examples which illustrate their application.

7. Vicinal Fragment Coefficients

7.1. Vicinal Groups and General Characteristics

As a class, vicinal fragment coefficients differ from those of geminal fragments in two ways. (1) Their nodal properties and magnitudes are much less varied, and (2) they are usually larger in magnitude. The latter is noteworthy since the vicinal positions are *farther* removed from the bond fragment, where most of the charge is concentrated.

The term "vicinal group" refers to the collection of vicinal fragments whose near hybrids (relative to the bond fragment) share the same carbon atom. Members of vicinal groups are classified as (1) *conjugate fragments*, those vicinal and coplanar with the bond fragment, or (2) *gauche fragments*, those vicinal and *non-coplanar* with the bond fragment. The character of the conjugate fragment will also be used to characterize the vicinal group itself. If the conjugate fragment is a *trans* primary alkyl CH fragment, then the vicinal group is a *trans* primary alkyl CH vicinal group. Note that this definition incorporates the relative orientation of the bond fragment and the vicinal group.

All vicinal fragment coefficients are largely determined by (1) the character of the vicinal group (or conjugate fragment character), and (2) the degree of unsaturation in the bond region *joining* the bond fragment and the vicinal group. Smaller coefficient variations accompany changes in the bond fragment atoms. The two coefficients of a given vicinal fragment have similar magnitudes, but opposite signs. In a particular vicinal group, the sign of the near (relative to the bond fragment) hybrid coefficient in the conjugate fragment is always the opposite of that in either (adjacent) gauche fragment. If the vicinal group character is *trans*, the coefficient of the near hybrid in the conjugate fragment is always negative and has a magnitude several times larger than those of the gauche fragments. In passing (from the *trans*) to the *cis* vicinal group, regular changes occur in *all* vicinal group coefficients. (1) The conjugate fragment coefficient magnitudes *decrease* by about 25%, (2) the *gauche* fragment coefficient magnitudes *increase* by 30–50% (and as a result generally become more than half as large as those of the conjugate fragment), and (3) *all* coefficient signs are reversed. In previous applications, these facts were found to be important for understanding the origins of hindered rotation [44–47].

7.2. Tabulation

Fragment coefficients are given in Tables 7 and 8 for all vicinal groups having *trans* character. Table 7 lists alkyl groups and Table 8 lists vinyl and ethynyl groups. Overall, there are three possibilities, corresponding to the vicinal group and bond fragment being joined by a single, double or triple bond region. These are designated by C–C, C=C and C≡C and consist of three, two and one fragment members, respectively. The C–C can occur for both the alkyl and vinyl groups, while C=C and C≡C necessarily occur for the vinyl and ethynyl groups, respectively. The dependences on the bond fragment type which occur in the C–C and C=C blocks are recorded by the column labels. The groups of rows specify the different vicinal groups, with the conjugate fragment given first. In

Table 7. Alkyl carbon LMO vicinal group coefficients

ALKYL CARBON VICINAL GROUP COEFFICIENTS										
C-C										
	CAH	CACA	CACV	CACE	CVH	CVC	BCVCV	BCECE	CIS	GC
CS	-440	-480	-465	-470	-400	-420	-490	-490	-90	20
H	430	430	415	415	390	390	480	490	-90	-15
CS	175	170	170	160	140	150	190	180	90	0
H	-160	-150	-150	-140	-130	-130	-180	-180	90	0
CS	110	125	125	115	95	90	130	120	90	0
C	-110	-110	-100	-95	-95	-85	-120	-120	90	0
CT	-320	-340	-300	-330	-265	-285	-335	-315	-90	0
CA	330	330	295	315	305	305	365	375	-90	0
CT	150	155	145	135	120	115	170	160	90	10
H	-150	-140	-140	-130	-125	-115	-160	-160	90	0
CT	90	105	105	95	70	65	120	110	90	10
C	-110	-110	-110	-100	-90	-80	-110	-110	90	0
CT	-310	-350	-310	-340	-270	-290	-340	-340	-30	15
CV	325	335	305	315	310	310	370	380	-60	-20
CT	150	155	145	135	120	115	170	160	50	0
H	-150	-140	-140	-130	-125	-115	-160	-160	30	0
CT	90	105	105	95	70	65	120	110	50	0
C	-110	-110	-110	-110	-90	-80	-110	-110	30	0
CT	-295	-310	-290	-300	-270	-275	-300	-300	-90	20
CE	300	290	275	280	290	290	320	330	-90	-15
CT	150	155	145	135	120	115	170	160	90	10
H	-150	-140	-140	-130	-125	-115	-160	-160	90	0
CT	90	105	105	95	70	65	120	110	90	10
C	-110	-110	-110	-110	-90	-80	-110	-110	90	0

the $C\equiv C$ block, all bond fragments have the same dependence and since each vicinal group has one member, the tabulation is quite simple. The fragment coefficients in each block are always given in the order (1) near hybrid, (2) far hybrid.

Consider the situation where a secondary CH vicinal group lies *trans* to a bond fragment between secondary and tertiary alkyl carbons. This case is covered by the CSH, CSH, CSC rows of the CACA column. Thus, the (*trans*) conjugate fragment coefficients are

$$CS = -480$$

$$H = 430$$

and the two gauche fragment coefficient sets are

$$CS = 170$$

$$H = -150$$

Table 8. Vinyl and ethynyl LMO vicinal group coefficients

VINYL CARBON VICINAL GROUP COEFFICIENTS										
C-C										
	CAH	CACA	CACV	CACE	CVH	CVC	BCVCV	BCECE	CIS	GC
CSV	-430	-475	-475	-455	-430	-455	-465	-455	-165	110
H	430	435	435	420	430	435	475	475	-130	-100
BCSV	130	130	130	130	130	130	160	160	75	-15
CV	-100	-100	-100	-100	-100	-100	-130	-140	90	-10
BCSV	-550	-595	-595	-575	-530	-575	-685	-630	-80	20
CV	540	530	530	515	540	540	660	650	-105	-30
BCSV	320	335	325	325	320	320	400	375	40	0
CV	-310	-310	-310	-300	-310	-310	-400	-385	50	-20
CSV	120	110	110	110	110	110	155	100	140	-35
H	-130	-110	-110	-110	-110	-110	-160	-140	120	40
BCE	-675	-735	-735	-715	-655	-715	-720	-750	-105	
CE	630	630	630	615	630	630	690	740	-95	
BCE	240	250	240	240	235	235	270	250	130	
CE	-230	-230	-230	-220	-230	-230	-255	-250	115	
C = C										
	CVH	CVC	GC		CVH	CVC	GC			
CPV	-555	-575	30		CSV	-380	-390	20		
H	525	515	-20		CV	385	375	-5		
CPV	335	335	-100		CSV	300	300	-85		
H	-300	-285	80		H	-300	-285	80		
CSV	-425	-435	30		CSV	-400	-410	30		
CA	415	405	-10		CE	380	370	-10		
CSV	300	300	-85		CSV	300	300	-85		
H	-300	-285	80		H	-320	-305	80		
ETHYNYL CARBON VICINAL GROUPS										
C ≡ C										
		CEH	CECA	CECV	CECE					
ALL BOND		-340	-265	-265	-250					
FRAGMENTS		345	275	275	250					

and

$$CS = 125$$

$$C = -110$$

for the CSH and CSC fragments, respectively.

Now, suppose we want the coefficients for a *primary* CH vicinal group lying trans to the bond fragment between secondary and tertiary alkyl carbons. These

are obtained from the GC column. The numbers shown there are (1) subtracted from the row entries when the near carbon of the vicinal group *in the table* is bonded to *more* carbon atoms, and (2) added to the row entries when the near carbon is bonded to fewer carbon atoms. That is, the procedure is dictated by the “geminal character” (GC) of the near carbon. In our case the near carbon we are considering is primary, whereas the near carbon shown in the table is secondary, so we subtract the numbers in the VF column from our previous numbers, giving

$$CP = -480 - 20 = -500$$

$$H = 430 + 15 = 445$$

and

$$CP = 170 - 0 = 170$$

$$H = -150 - 0 = -150$$

for the primary CH group conjugate and gauche coefficients, respectively. Note that the CSC gauche fragment is not relevant to this calculation because there are no CPC vicinal fragments. To illustrate the other case, suppose a tertiary CH conjugate fragment lies *trans* to the same bond fragment. This time we have (since the near carbon of the vicinal group in the table is bonded to fewer carbon atoms),

$$CT = -480 + 20 = -460$$

$$H = 430 - 15 = 415$$

and

$$CT = 125 - 0 = 125$$

$$C = -110 - 0 = -110$$

for the *trans* and gauche fragment coefficients. Note that here the CSH gauche fragment coefficients are irrelevant.

The usage of the GC column is slightly different when the near carbon of the vicinal group is a vinyl carbon and the vicinal group and the bond fragment are joined by a single bond region (Table 8). Suppose we take a secondary vinyl CH group *trans* to a CSVCS bond fragment as an example. (The CSV carbon of the bond fragment is nearest to the vicinal group.) The coefficients are obtained from the C–C block, rows CSVH and BCSVCV, column CVC. They are,

$$CSV = -455$$

$$H = 435$$

for the conjugate fragment, and

$$BCSV = 130$$

$$CV = -100$$

for *both* gauche fragments. The necessarily different usage of the GC column can be appreciated by two observations: (1) a CPVH vicinal group cannot be joined to any bond fragment by a *single* bond region, and (2) tertiary vinyl CH fragments don't exist. Thus, the numbers in the GC column describe the coefficient changes that occur in passing from a CSVH vicinal group to a CTVC vicinal group. For example, the coefficients for a CTVCP vicinal group *trans* to a CSVCS bond fragment are calculated from those of the previous CSVH case as

$$\text{CTV} = -455 + 110 = -345$$

and

$$\text{CP} = 435 - 100 = 335$$

$$\text{BCTV} = 130 - 15 = 115$$

$$\text{CV} = -100 - 10 = -110.$$

Cis vicinal fragments in the CC blocks are obtained from the *trans* coefficients and the column labeled "*Cis*". For example, suppose we return to the situation where the secondary CH vicinal group lies *trans* to a bond fragment between secondary and tertiary alkyl carbons. We get the *cis* vicinal group coefficients by taking the numbers in the *Cis* column, adding them to the *magnitudes* of the *trans* group coefficients, and then reversing the signs relative to the original coefficients. Thus,

$$\text{CS} = (480 - 90) = 390$$

$$\text{H} = -(430 - 90) = -340$$

for the *cis* conjugate fragment and

$$\text{CS} = -(170 + 90) = -260$$

$$\text{H} = (150 + 90) = 240$$

and

$$\text{CS} = -(170 + 90) = -260$$

$$\text{C} = (110 + 90) = 200$$

for the two corresponding gauche fragments of the *cis* CSH vicinal group.

Cis fragments are explicitly listed in the C=C block because both vicinal fragments are necessarily coplanar with the bond fragment. The *trans* fragment will be used to characterize the vicinal group, and therefore its coefficients are always listed first. Consider a CSVCTV vicinal fragment *trans* to a CPVH bond fragment. This case is covered by the CSVCV and CSVH rows of the CVH column. The *trans* fragment coefficients are

$$\text{CSV} = -380$$

$$\text{CTV} = 385$$

and the *cis* fragment coefficients are

$$\text{CSV} = 300$$

$$\text{H} = -300.$$

8. Third and Fourth Neighbor Fragment Coefficients

8.1. Third Neighbor Groups

Third neighbor groups are the fragments common to a far carbon of the vicinal fragment. Four cases are observed, two for conjugate fragment far carbons, and two for gauche fragment far carbons. The former, termed T and C, correspond to situations where the conjugate and bond fragments lie *trans* or *cis*, respectively. The latter, termed S and S', correspond to situations where the dihedral angle between the bond and conjugate fragments is about 110° and 55°, respectively. The bond region skeletons defining these are shown on Fig. 3. On the figure, the bond fragment is labeled B. F labels the third neighbor fragment lying in the plane defined by two bond regions. The first bond region is the one connecting the third neighbor group to the near vicinal carbon (bond region *r* on the figure) and the second is the one connecting the vicinal group to the near carbon of the bond fragment (bond region *r'* on the figure). In the case of S and S', there is asymmetry about this plane. The third neighbor fragment on the same side of the plane as the bond fragment is called the CX(+) third neighbor fragment, while the one on the other side is called the CX(-) third neighbor fragment.

The coefficients for the third neighbor fragments are listed in Table 9. There are four divisions within the table corresponding to the four observed cases. The column headings in each division describe the F fragment dependences. F* means that the F fragment lies *cis* to the previously defined bond region *r'*. The F fragment dependence in the T and C sections differs from that in the S and S' sections.

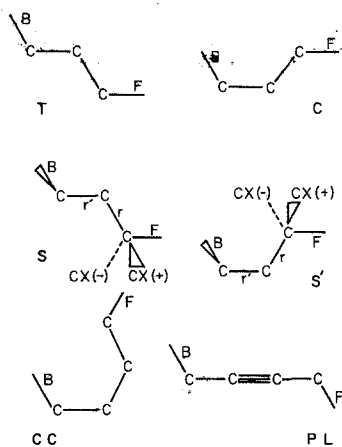


Fig. 3. Third and fourth neighbor fragment orientations. B labels the bond fragment. The fragments F, *r*, and *r'* are discussed in the text

Table 9. Third and fourth neighbor LMO group coefficients

THIRD NEIGHBOR GROUP COEFFICIENTS										
-T-										
F =	H	C	2	3	H*	C*	2*	3*		
F	130 -130	110 -110	150 -165	200 -200	-25 45	0 25	-40 80	-90 115		
CH	-15 0	-5 20	10 0		80 -80	70 -70	90 -90	140 -140		
CC, 2, 3	0 0	0 10	-85 80	-30 30	60 -60	50 -60	60 -70	80 -90		
-C-										
F =	H	C	2	3	H*	C*	2*	3*		
F	-100 110	-100 110	-100 110	-100 110	-65 40	-65 40	-65 40	-65 40		
CH	-40 20	-40 20	-40 20		-40 45	-15 30	-15 30			
CC, 2, 3	-5 30	-5 30	-5 30	10 -10	-40 45	-15 30	-5 30	10 -10		
-S-										
F =	H	C	CVY	2	3	H*	C*	CVY*	2*	3*
F	-50 70	-60 70	80 -70	-60 70	-85 95	-10 -10	-10 -10	10 -10	-10 -10	-10 -10
C	-40	-30	-20	-30	30	0	0	0	0	0
X(+)	40	0	30	0	-50	-15	-25	10	-25	-10
C	20	20	-20	20	0	-80	-80	0	-80	-105
X(-)	0	0	30	0	25	110	110	10	110	135
-S'-										
		F	F*			F	F*			
BCVCV BOND FRAGMENTS		F	-80 80	10 -10			F	-40 10	10 -10	
		C	-45	0	OTHER BOND FRAGMENTS		C	-30	30	
		X(+)	5	-20			X(+)	30	-30	
		C	15	80			C	-60	70	
		X(-)	0	-90			X(-)	90	-50	
FOURTH NEIGHBOR GROUP COEFFICIENTS										
-CC-					-PL-					
F =	CX	CVX	CX*	VVX*	F =	CX	CX*			
F	-180 150	-170 155	-190 160	-180 190	F	65 -135	-60 90			
C	25	-190	10	-180	C	-35	50			
Y	-15	200	0	80	Y	70	-80			

The vinyl carbon single bond F fragments in the S section (Columns CVY and CVY*) behave differently from the other single bond F fragments (Columns H, C, H*, and C*). In the S' section there is a bond fragment dependence only. That is, the heading BCVCV Bond Fragments means that the numbers to the immediate right apply when the bond fragment is a double banana bond, and the heading Other Bond Fragments precedes the numbers that apply in all other cases. The row labels characterize the F fragment and those fragments geminal to it. The order of tabulation is near hybrid, far hybrid. Thus, the row labels CH and CC,

2, 3 in the T and C sections characterize all carbon-hydrogen fragments and singly, doubly and triply bonded carbon-carbon fragments, respectively. The row labels CX(+) and CX(-) in the S and S' sections mean that only the orientation of the fragments relative to the plane defined by r and r' is important, i.e. we needn't specify CH(+), CH(-), CC(+), etc.

8.2. Fourth Neighbor Groups

Fourth neighbor groups consist of the fragments common to far carbons of third neighbor groups. Only two cases are found to have coefficients large enough to warrant classification. The first, CC, occurs when all bond regions between the bond fragment and the fourth neighbor fragment lie coplanar and *cis*. The second, PL, occurs when the near carbon of the *third* neighbor fragment is an ethynyl carbon. Both are shown on Fig. 3. The B labels the bond fragment and the F labels the fourth neighbor fragment coplanar to B.

Coefficients for fourth neighbor fragments are likewise listed in Table 9. The column labels characterize the F fragment and the row labels distinguish the F fragment from the other fourth neighbor fragments. The order of tabulation is near hybrid, far hybrid. F* means that F lies *cis* to the conjugate fragment. Other than the geometric dependence which defines the fourth neighbor group itself, there is virtual insensitivity. In particular there is no dependence on the far atom of F (X in the column labels) or the far atom of its neighbor fragments (Y in the row labels). However, differences are observed in the CC section according to whether the near carbon of F is alkyl or ethynyl (Columns CX and CX*) or whether it is vinyl (Columns CVX and CVX*).

9. Examples of Model LMO's

Sample LMO's obtained from molecular fragment coefficients are compared with exact LMO's in Table 10. The row labels define the atomic basis functions as in the previous Tables 1 and 2. The first of the two sets of column labels specifies the molecule from which the exact LMO was taken. Two columns of coefficients are given below the molecule label and these contain the exact LMO and the corresponding model LMO. The exact LMO is always given first and the number above it is the same as its column label in the previous Table 2. The model LMO's are headed by the label M. For example, the first column in Table 10 contains LMO Number 4 in butane (from the previous Table 2) and the second column contains its molecular fragment model.

Overall agreement is excellent and significantly better than shown for the SMO's and PMO's in Table 1. Deviations between exact and model coefficients are less than 1% as large as the bond fragment coefficients. This is typical of the modelling scheme. If higher accuracy is needed, a more detailed model can be used. This would be preferred for constructing model geometry, model LMO molecules via a computer program. However, the present model is accurate enough for most purposes and describes the situation in a way that reveals the important influences.

Table 10. Comparison of accurate and model LMO's

	BUTANE		ISOBUTANE		BUTENE		BUTADIENE		BUTYNE		VINYL ACETYLENE	
	12	M	4	M	5	M	4	M	10	M	3	M
1	7069	7066	7019	7021	384	380	399	400	-24	-20	-35	-30
2	173	150	175	180	326	325	309	340	388	390	352	350
3	86	90	-40	-40	7062	7061	7034	7035	-35	-15	20	-15
4	86	90	-40	-40	-56	-60	-9	-60	-35	-15	7037	7041
5	7040	7036	7039	7041	210	220	112	115	7127	7131	-75	-15
6	-82	-80	72	75	382	375	370	375	52	235	310	310
7	140	140	144	145	7005	7011	6984	6985	22	20	19	20
8	140	140	144	145	47	50	53	50	52	20	7010	7011
9	140	145	61	60	-485	-490	95	95	6969	6971	9	20
10	53	60	-496	-500	82	95	152	155	13	40	66	70
11	140	145	174	170	125	130	403	400	-313	-315	-246	-240
12	-346	-340	162	170	191	190	-687	-685	154	170	-413	-415
13	148	130	61	60	-77	-80	98	0	159	170	546	550
14	334	330	-496	-500	-38	-45	-9	0	382	375	-34	0
15	-18	-15	162	170	-119	-120	-409	-400	-14	-15	-47	0
16	-18	-15	174	170	27	15	664	660	-14	-15	-548	-545
17	-20	-25	-34	-30	1	5	0	10	161	130	434	435
18	-11	-15	-16	-5	10	10	10	0	-182	-160	260	260
19	-11	-15	-24	-25	-15	-10	-7	-10	-186	-160	12	0
20	-29	-25	-24	-25	473	480	-160	-160	18	20	60	0
21	-29	-25	442	445	-177	-180	-85	10	18	20		
22	-145	-140	-150	-150	79	80	17	0	-154	-130		
23	-145	-140	-138	-150	3	5						
24	-134	-130	442	445	6	0						
25	20	20	-138	-150								
26	20	20	-150	-150								

	BUTANE		ISOBUTANE		BUTENE		BUTYNE		BUTENE		BUTYNE	
	4	M	7	M	2	M	11	M	12	M	1	M
1	-144	-150	-69	-65	7004	7011	6874	6786	-561	-575	352	345
2	6971	6976	6905	6911	280	280	7228	7231	331	335	-349	-340
3	139	135	-69	-65	-267	-265	8	5	-62	-60	-285	-290
4	139	135	-69	-65	-267	-265	8	5	-62	-60	-285	-290
5	-9	0	-3	-5	-425	-425	-18	-15	7014	7016	-292	-290
6	-322	-320	-465	-460	299	300	-264	-265	314	295	7108	7111
7	145	140	176	175	-131	-120	-334	-335	-235	-240	-36	-40
8	145	140	176	175	-131	-120	-334	-335	-235	-240	-37	-40
9	-6	-5	-3	-5	70	70	-345	-345	186	195	82	90
10	343	330	176	175	417	415	279	275	7016	7016	6956	6961
11	-6	-5	176	175	0	0	12	0	-114	-115	-68	-50
12	105	110	-465	-460	70	70	23	0	186	195	160	160
13	-47	0	-3	-5	52	0	23	0	-453	-440	160	160
14	-111	-110	176	175	-2	0	-18	0	156	150	94	95
15	11	0	-465	-460	26	25	10	0	62	60	159	160
16	11	0	176	175	-2	0	10	0	156	150	159	160
17	7138	7136	7122	7121	7074	7066	-24	0	504	515	-483	-490
18	-132	-125	441	445	-274	-270	-24	0	-290	-285	2	0
19	-131	-125	-161	-160	-295	-300	-24	0	-187	-185	2	0
20	-156	-150	-161	-160	-78	-70	-13	0	-23	-25	-135	-140
21	-156	-150	-161	-160	-78	-70	-12	0	-23	-25	-135	-140
22	21	20	-161	-160	-38	0	35	0	411	405	432	430
23	21	20	441	445	7	0			-136	-130		
24	59	0	-161	-160	7	0			-136	-130		
25	-14	0	441	445								
26	-14	0	-151	-160								

10. Canonical and Virtual Orbitals

10.1. Introduction

These orbitals have several properties interesting enough to call for some way of determining them from the localized orbitals. Chief among these are that they approximate the selection rules for electronic transitions and provide estimates of the lower excited states of the electronic spectrum. To determine them, all that is actually needed is the knowledge that the CMO's diagonalize the Fock matrix; but this implies that they are symmetry orbitals, i.e., carrier functions for the irreducible representations of the molecule's point group. This can be exploited in many cases to shorten their computation.

10.2. Virtual Orbitals

The virtual, or unoccupied, MO's (VMO's) are those produced in any closed-shell LCAO-SCF-MO calculation having more basis functions than electron pairs. They can be used, in canonical form, to approximately describe the electronic spectrum and, in any form, to calculate the so-called polarizabilities useful in the applications of perturbation theory to MO wave functions [36–42]. It is therefore desirable to compute them as simply as possible as well as having some way of casting them in canonical form. The latter will be given in the next section.

The simplest way to calculate VMO's from LMO's makes use of the notion of antibonding. Let c and c' be bond fragment coefficients for hybrids h and h' , respectively, in some LMO λ . The INDO antibonding virtual localized orbital is defined

$$\lambda' = (c'h - ch')/(c^2 + c'^2)^{1/2} \quad (36)$$

and, in the paraffins, are in one-to-one correspondence with the occupied LMO's. The λ' are mutually orthogonal, but only approximately orthogonal to the occupied space. To make them so orthogonal, some process, e.g. Gram-Schmidt orthogonalization should be used which doesn't change the occupied space, as this is the one accurately modeled. When this has been done, one has a set of virtual LMO's (VLMO's).

10.3. Canonical Orbitals

In the present context, one should also determine CMO's for the occupied and virtual spaces without altering the former. Thus, let the λ 's be LMO's or VLMO's and let

$$\lambda_i = \sum_{\mu} T_{i\mu} f_{\mu} \quad (37)$$

be their expansions in symmetry orbitals. Using standard projection operators [48] P_{μ} for each irreducible representation, one can project the set

$$\mu_i = P_{\mu} \lambda_i = T_{i\mu} f_{\mu} \quad (38)$$

from each λ , and by Gram-Schmidt orthogonalization, obtain a set of N_{μ} linearly independent symmetry MO's μ within the space in question. Since the Fock operator depends only on the occupied space, it is known and its matrix elements, $F(\mu; ij)$ in the μ basis can be calculated. The CMO's of symmetry μ are the eigenfunctions of this N_{μ} by N_{μ} matrix, and repeating the process for all symmetries produces all of the CMO's.

When a given symmetry occurs only once, N_{μ} is unity and the μ_i is the CMO. At the other extreme, when there is no symmetry, there are no short cuts and the full Fock matrix in the occupied and/or virtual spaces must be diagonalized. In this situation, one may consider another option, especially for smaller molecules, namely, diagonalizing the Fock matrix in the atomic basis and simultaneously obtaining the occupied and virtual CMO's.

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